1 A particle rests on a smooth, horizontal plane. Horizontal unit vectors **i** and **j** lie in this plane. The particle is in equilibrium under the action of the three forces $(-3\mathbf{i} + 4\mathbf{j})$N and $(21\mathbf{i} - 7\mathbf{j})$N and **R**N.

(i) Write down an expression for **R** in terms of **i** and **j**. [2]

(ii) Find the magnitude of **R** and the angle between **R** and the **i** direction. [4]

2 The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where **i** and **j** are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

(iii) Find the coordinates of the point where the particle is moving at 45° to both Ox and Oy. [3]

3 The vectors **p** and **q** are given by

$$\mathbf{p} = 8\mathbf{i} + \mathbf{j} \text{ and } \mathbf{q} = 4\mathbf{i} - 7\mathbf{j}.$$

(i) Show that **p** and **q** are equal in magnitude. **[3]**

(ii) Show that **p** + **q** is parallel to $2\mathbf{i} - \mathbf{j}$. **[2]**

(iii) Draw **p** + **q** and **p** − **q** on the grid.

Write down the angle between these two vectors. **[3]**

4 In this question, **i** is a horizontal unit vector and **j** is a unit vector pointing vertically upwards.

A force **F** is $-\mathbf{i} + 5\mathbf{j}$.

(i) Calculate the magnitude of **F**.

Calculate also the angle between **F** and the upward vertical. **[4]**

Force **G** is $2a\mathbf{i} + a\mathbf{j}$ and force **H** is $-2\mathbf{i} + 3b\mathbf{j}$, where a and b are constants. The force **H** is the resultant of forces $4\mathbf{F}$ and **G**.

(ii) Find **G** and **H**. **[4]**

5 The resultant of the force $\begin{pmatrix} -4 \\ 8 \end{pmatrix}$ N and the force **F** gives an object of mass 6 kg an acceleration of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ m s^{-2}.

(i) Calculate **F**. **[4]**

(ii) Calculate the angle between **F** and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. **[2]**

6 The force acting on a particle of mass 1.5 kg is given by the vector $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ N.

(i) Give the acceleration of the particle as a vector. [2]

(ii) Calculate the angle that the acceleration makes with the direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [2]

(iii) At a certain point of its motion, the particle has a velocity of $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m s^{-1}. Calculate the displacement of the particle over the next two seconds. [3]

7 A force **F** is given by $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$ N, where **i** and **j** are horizontal unit vectors east and north respectively.

(i) Calculate the magnitude of **F** and also its direction as a bearing. [3]

(ii) **G** is the force $(7\mathbf{i} + 24\mathbf{j})$ N. Show that **G** and **F** are in the same direction and compare their magnitudes. [2]

(iii) Force $\mathbf{F}_1$ is $(9\mathbf{i} - 18\mathbf{j})$ N and force $\mathbf{F}_2$ is $(12\mathbf{i} + q\mathbf{j})$ N. Find q so that the sum $\mathbf{F}_1 + \mathbf{F}_2$ is in the direction of **F**. [2]